Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

rily equally probable. For instance, a finite number of units in the numerator contains, as individual units, many *concentric* circles; whereas it may easily be shown that the chance that there will be *two* concentric circles in a finite number of units of the denominator, is infinitely small.

Mr. Adcock requests us to make the following corr. in his solution of 286: "For number of positions use the number of combinations instead of the number of permutations. For instance, for $\pi^3m^3r^6$ read, $\frac{1}{6}\pi^3m^3r^6$."—Ed.]

---

308. "Evaluate

$$\int_0^{\frac{a}{2}} dx \sin\left[x(a-x)\right] \text{ and } \int_0^{\frac{a}{2}} dx \cos\left[x(a-x)\right]$$

by a rapidly converging series."

SOLUTION BY R. J. ADCOCK, ROSEVILLE, ILL.

Since $\sin(ax-x^2) = ax-x^2-\frac{(ax-x^2)^3}{1.2.3}+\frac{(ax-x^2)^5}{1.2.3.4.5}-$&c., and by p'ts,

$$\int(a-x)^n x^n dx = -\frac{(a-x)^{n+1}x^n}{n+1}-\frac{n(a-x)^{n+2}x^{n-1}}{(n+1)(n+2)}-\frac{n(n-1)(a-x)^{n+3}x^{n-2}}{(n+1)(n+2)(n+3)} -\&\text{c.}+C.$$

$$\therefore \int_0^{\frac{1}{2}a}(a-x)x\,dx = -\tfrac{1}{2}(a-x)^2x-\tfrac{1}{6}(a-x)^3+C = \tfrac{1}{12}a^3, \text{ and}$$

$$\int_0^{\frac{1}{2}a}(a-x)^3x^3dx = \frac{a^7}{280}, \int_0^{\frac{1}{2}a}(a-x)^5x^5dx = \frac{a^{11}}{5324}; \text{ hence}$$

$$\int_0^{\frac{1}{2}a} dx \sin\left[x(a-x)\right] = \frac{a}{12}-\frac{a^7}{1680}+\frac{a^{11}}{106480}-\&\text{c.},$$

which converges rapidly for values of a not greater than one.

SOLUTIONS OF PROBLEMS IN NUMBER THREE.

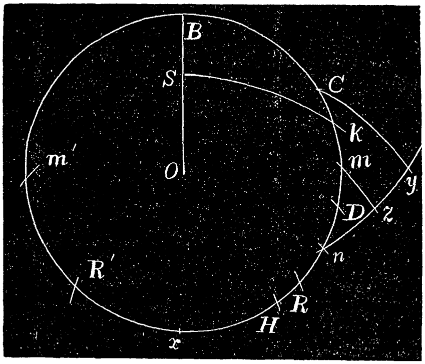
Solutions of problems in No. 3 have been received as follows:

From Geo. H. Bangs, 311; Alex. S. Christie, 310; Prof. W. P. Casey, 309, 310, 311; Geo. Eastwood, 310; Prof. E. J. Edmunds, 309, 310; H. Heaton, 310, 311, 312; Lieut. Jas. M. Ingalls, 312; Prof. J. H. Kershner, 309, 310, 311, 312 [Prof. Kershner should have been credited in No. 3 for a solution of 304, which was omitted by an oversight]; W. L. Marcy, 311; Prof. D. J. Mc Adam, 311; P. Richardson, 311; Prof. J. Scheffer, 310, 311, 312; J. L. Schock, 312; Prof. De Volson Wood, 311.

---

309. "In a given circle, find the vertices of the inscribed square, pentagon, octagon, and decagon by using the dividers alone."

SOLUTION BY PROF. W. P. CASEY, SAN FRANCISCO, CAL.



Let O be the centre of the given circle. Take xn, nC, CB, each = radius, and with B as centre and radius Bn, describe the arc ny, and with x as centre describe the arc Cy; also with x as centre and radius Oy describe the arc $m'mz$, and with the same centre and radius Oz, describe the arc D and with CD as radius and centre x describe the arc RR'. The points x, m, B, m' are the vertices of the inscribed square, and m', R', x, R, m, &c., are vertices of the inscribed octagon, as may be easily proved.

Granting that a line may be divided into two equal parts by using the dividers alone, then we can divide the radius OB in extreme and mean ratio by the dividers, and if OS be the greater segment, take x as centre and with radius xS describe the arc SK, then will BK be a side of the inscribed pentagon. Again, with radius OS and centre at x describe the arc H, then is xH a side of the inscribed decagon.

---

310. "Required the locus of vertices of a right angled spherical triangle whose legs pass through two fixed points given on the surface of the sph."

SOLUTION BY A. S. CHRISTIE, COAST AND GEODETIC SURV., WASH., D. C.

With the center of the sphere as origin, let α and β be the vectors of the given points and ρ the vector through a point of the locus. Then

$$0 = SV\alpha\rho\, V\beta\rho = S\alpha\rho S\beta\rho - \rho^2 S\alpha\beta,$$

a cone of the second degree of which the cyclic planes are

$$S\alpha\rho = 0, \quad S\beta\rho = 0.$$

Hence the locus is the spherical conic

$$0 = S\alpha\rho\, S\beta\rho - S\alpha\beta, \quad T\rho = T\alpha = T\beta = 1.$$

---

311. "A uniform circular plate is placed with its centre upon a prop, to find at what points on its circumference three given weights p, q, r must be attached that it may remain at rest in a horizontal position."

SOLUTION BY PROF. DE VOLSON WOOD, HOBOKEN, N. J.

Take the origin of coordinates at the centre of the circle, the axis of x

passing through p; x, y, the coordinates of q, x_1, y_1, of r, and R the radius of the circle. Then by moments we have

$$pR = qx + rx_1,$$

$$qy = ry_1;$$

and from the circle

$$R^2 = x^2 + y^2 = {x_1}^2 + {y_1}^2.$$

Solving we find

$$x = \tfrac{1}{2}R\frac{r^2+p^2-q^2}{pr}, \quad y = \pm R\sqrt{\left(1 - \frac{(r^2+p^2-q^2)}{4p^2r^2}\right)};$$

in which if y be real, the solution will give two points, but if imaginary the problem will be physically impossible. Similarly values may be found for x_1 and y_1.

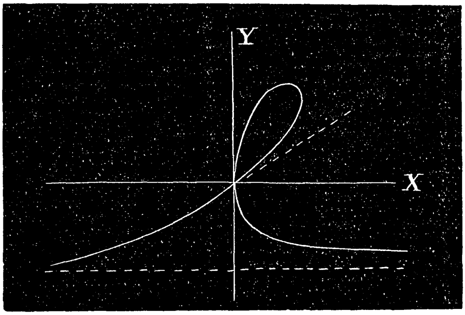
312. "To find the area of the loop of the curve $y^3 + x^2y - axy + bx^2 = 0$."

SOLUTION BY J. L. SCHOCK, CADET MID'N, U. S. N. A.

The equation $y^3 + x^2y - axy + bx^2 = 0$ (1), gives

$$y = \frac{axy - bx^2}{x^2+y^2}, \text{ or at the limit } y = \left.\frac{axy-bx^2}{x^2+y^2}\right]_{y=0} = -b.$$

Hence $y = -b$ is an asymptote to the curve. $x(ay - bx) = 0$ are the eq's of the tangents at the origin since these terms contain all the lower powers. The curve then evidently is:—



$$A = \tfrac{1}{2}\int r^2 d\theta.$$

The limits of integration for polar coordinates evidently are

$$\tan^{-1}\frac{b}{a} \text{ and } \frac{\pi}{2},$$

as shown in the curve. Transforming (1) to polar coordinates,

$$r = \frac{\cos\theta}{\sin\theta}\left[a\sin\theta - b\cos\theta\right]. \tag{2}$$

$$\therefore \text{Area of loop} = \tfrac{1}{2}\int_{\tan^{-1}\frac{b}{a}}^{\frac{1}{2}\pi} \frac{\cos^2\theta}{\sin^2\theta}\left[a\sin\theta - b\cos\theta\right]^2 d\theta. \tag{3}$$

The integral of (3) is

$$\tfrac{1}{4}(a^2-3b^2)\theta + \tfrac{1}{4}(a^2-b^2)\sin\theta\cos\theta - ab(\log\sin\theta + \tfrac{1}{2}\cos^2\theta) - \tfrac{1}{2}b^2\cot\theta.$$

Substituting the limits we have

$$\frac{1}{4}\left[\left(a^2-3b^2\right)\left(\frac{\pi}{2} - \tan^{-1}\frac{b}{a}\right) + ab\left(2\log\frac{b^2}{a^2+b^2} + 3\right)\right], \text{ or}$$

$$\frac{1}{4}\left[\left(a^2-3b^2\right)\left(\cot^{-1}\frac{b}{a}\right) + ab\left(2\log\frac{b^2}{a^2+b^2} + 3\right)\right],$$

the required area.